

A Visual History of Jean Perrin's Brownian Motion Curves

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In sum the science of drawing consists in instituting relations between curves and straight lines. A painting containing only curves or straight lines would not express existence. (En somme la science du dessin consiste à instituer des rapports entre les courbes et les droites. Un tableau qui ne contiendrait que des droites ou des courbes n'exprimerait pas l'existence.)

ALBERT GLEIZES AND JEAN METZINGER, *Du "Cubisme"* (1912)

A sheet of squared paper on which three broken lines have been drawn. A connect-the-dots game gone slightly awry, with no pattern obviously recognizable. No scale is inscribed that might provide clues about the size and nature of the object or phenomenon represented here. No indications on the procedure involved in the production of this two-dimensional abstraction. No numbers, letters, or symbols to tell the viewer how to hold the figure, or in what direction the lines run; indeed, its author (or perhaps was it the publisher's initiative?) occasionally published it sideways (fig. 6.1).¹

Yet show this image to a physicist or a mathematician and the response will be immediate: *this is Brownian motion*.

This image, published for the first time in September 1909 by French physical chemist Jean Perrin (1870–1942),² has acquired iconic status in the physical sciences. It was and is still perceived as an experimental confirmation and a visual equivalent of Albert Einstein's theoretical demonstration, in a paper of 1905, of "the reality of atoms and molecules, of the kinetic theory of heat, and of the fundamental part of probability in the natural laws."³

Yet Einstein's publications on Brownian motion do not feature any images, nor did he suggest that the phenomenon should be represented in this way. In fact, until proven wrong, Einstein even doubted that the methods he suggested for measuring Brownian motion could be realized experimentally: "I would have thought such a precise study of Brownian motion impossible to realize," he wrote in admiration to Perrin in November 1909.⁴

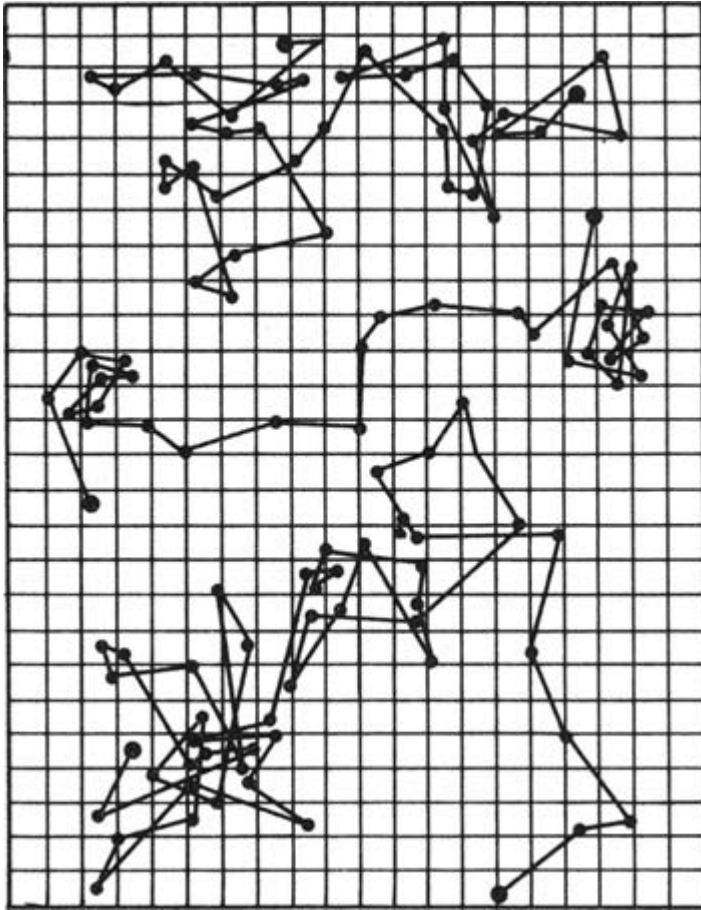


FIGURE 6.1. Jean Perrin, "Mouvement brownien et réalité moléculaire," *Annales de chimie et de physique* ser. 8, 18 (1909): 81.

Einstein's and Perrin's Brownian motion work is justly famous for raising a number of issues central to the epistemology and historiography of the physical sciences, in particular, related to the nature of evidence, the relationship between theory and experiment, and realism.⁵

Rather than investigating the detailed ways in which a perfect fit between Perrin's experiments and Einstein's theory was realized, this paper explores the gap between Einstein's formulas and Perrin's image, a gap that stretched across four years and different scientific cultures but was subsequently erased when the image collapsed onto the formula it represents. What happens when we pry apart the formula from its representation? How did this image

come both to encapsulate and help permanently establish a new way of seeing and understanding Brownian motion?

Certainly, Perrin's work took place within an epistemological economy structured by the twin categories of theory and experiment. When he wrote that his Brownian motion project was explicitly devised to serve as a crucial experiment to test the validity of the kinetic theory and the atomic hypothesis, Perrin adhered to the ruling epistemology of the laboratory sciences since the second half of the nineteenth century, in which observation played an epistemologically subordinate role.

Nonetheless, Perrin's work constitutes a milestone in the history of observation in the physical sciences, having established the existence of that most famous of all unobservables, the atom—though in fact neither atoms nor molecules were ever actually observed. A close look at his work reveals that observation, as a practice, as a skill, and as a product occupy a central place in Perrin's project. By focusing the attention on the techniques, skills, and resources involved in Perrin's practical laboratory work, this chapter not only shows how much takes place between the development of a theory and its experimental verification, and in particular, the role played by visual representations in the production of evidence; it also serves as a good reminder of the continuing importance of creative skill and technical ingenuity in the experimental sciences, broaching several of the issues developed in part 3 below, "Techniques." A parallel reading of this chapter together with Mary Terrall's chapter on Réaumur's observation of frogs in this volume brings out especially vividly, beyond the obvious differences, surprisingly similar concerns with ways of presenting the interaction of observers and their objects and the social organization and bodily disciplines of virtuoso observation. Finally, by uncovering the interdisciplinary dialogue that was determinant in the elaboration and subsequent appropriations of this image, this chapter shows how Perrin's work was embedded in the cultural and scientific fabric of his times. Tracing the history of this image, it shows how a virtual community was created through the making of Perrin as an observer and the making and the reception of his observations of Brownian motion.

"Mise en Observation"

Figure 6.1 appeared for the first time in the September 1909 issue of the *Annales de chimie et de physique*. As Perrin's laboratory notebooks of the time testify (fig. 6.2), this image was a composite picture of three drawings made earlier that year by Perrin during a series of experiments carried out together

with his student Dabrowski in the small laboratory for physical chemistry Perrin had set up at the turn of the century in an attic of one of the Sorbonne buildings.

In 1898, shortly after completing studies in physics and chemistry at the École Normale Supérieure with a Ph.D. showing that cathode rays were negatively charged (and therefore consisting of streams of particles, later to be named electrons), Perrin had begun teaching physical chemistry at the Sorbonne. He had been entrusted by the recruiting commission, in the person of Henri Poincaré, with the task of “naturalizing” on French soil the work of the mainly German pioneers and of bringing Gallic precision to the dynamic new field.⁶ Taking up the challenge, Perrin recast in his own words contemporary discussions on the relationship between thermodynamics and mechanics, propagating his views in a textbook, *Traité de chimie physique. Les principes* that appeared in 1903.⁷ There, but also in his teaching, popular lectures, and articles,⁸ he stood up as a staunch advocate of atomism, a minority position in the French scientific community—though one that was well represented at the École Normale Supérieure and in the circle of scientists he interacted with on a daily basis, including his teacher Aimé Cotton, Marie and Pierre Curie, and Paul Langevin.

In 1903, once his laboratory had been fitted out, Perrin launched a series of experiments on the electrical properties of colloid solutions, suspensions of submicroscopic particles, that increasingly attracted the interest of chemists and biologists in these years, and on which Cotton was working with a colleague at the Institut Pasteur, Henri Mouton.⁹ From there, Perrin moved on in early 1908 to the study of the Brownian motion of colloid particles.

Figure 6.1 was a product of one of the very last experiments within this new project. Perrin and Dabrowski had prepared what they referred to as their emulsion by bringing mastic, extracted from the bark of *Pistacia lentiscus* from the island of Chios and commonly used in the production of varnishes, into contact with methyl alcohol, obtaining a solution floating above a sticky insoluble residue. When diluted extensively, the solution became white as milk, in fact a suspension of spherical granules of varying sizes. Perrin and Dabrowski then subjected this emulsion to a series of “fractioned centrifugations” to obtain a suspension of grains of identical sizes. For this experiment they selected grains of a radius of 0.52 micrometers.¹⁰

In order to observe his emulsions, Perrin usually placed a drop of the suspension in a cavity about one-tenth of a millimeter deep, created by gluing a glass plate in which a wide hole had been bored onto an object slide. The cavity was then covered with another glass plate and sealed. This preparation could be used for several days or weeks.¹¹ For this particular experiment, the

stability of the liquid's viscosity (and hence its temperature) was essential, so Perrin and Dabrowski immersed both the cell containing the drop and the microscope objective in a water-filled tank. The temperature was regularly measured by dipping a thermometer close to the microscope objective. On the notebook page shown in figure 6.2 the temperature is indicated, "23°,25" (23,25° C).

These procedures—the selection of the grains, enclosing the emulsion, and setting up the microscope for optimal conditions of visibility—constituted, with slight variations involving different types of microscopes and different sizes and types of grains, the starting point for all of Perrin's Brownian motion experiments. They were described by him as the "mise en observation" of his emulsions, the setting up of the conditions under which the Brownian motion of colloid particles was to be observed and produce fruitful insights.¹² Perrin relied mostly on techniques learned or developed in previous years. As he later wrote, "the study of colloids had familiarized me with the observation of Brownian motion."¹³ It is noteworthy that many of these techniques were borrowed or adapted from the biologist's arsenal, from centrifugation, "as one does to separate the red cells from blood serum," to the use of dissection needles and plates engraved with a grid to help count cells in solutions, not to mention the microscope and camera lucida, standard microbiologist's devices.¹⁴ The cheapness and ready availability of these instruments were certainly an advantage in a modestly endowed laboratory such as Perrin's. Moreover, they testify to Perrin's close acquaintance, perhaps through Mouton, of biological techniques and their creative adaptation for physical-chemical investigation.

The majority of Perrin's observations concerned the behavior of large numbers of particles, for instance, when they reached equilibrium in the emulsion, their concentration decreasing with altitude. In the particular experiment discussed here, Perrin and Dabrowski were interested instead in measuring the motion of *individual* particles.

The broken line at the center of figure 6.2 represents the motion of a single particle over a period of 20 minutes ("20 minutes de pointés"). In the published version (fig. 6.1), 16 divisions of the grid correspond to 50 micrometers. This is a simplified representation of the more complex trajectory followed by the particle, obtained by marking its position at regular intervals of time, here every 30 seconds. The trajectory is also simplified in the sense that it is a projection on a two-dimensional plane of the three-dimensional motion of the particles in the liquid. For this, Perrin attached a camera lucida to his microscope, enabling the simultaneous visualization of the particles swimming in the liquid and of his sheet of paper. While one man stationed

in front of a chronometer called out the signals every 30 seconds, the other stood at the microscope eyepiece, following the motion of a particle and noting its position on paper when the signal was called. Perrin and Dabrowski regularly swapped positions, presumably to compensate for personal idiosyncrasies in each scientist's observing technique. The dots on the sheet were then numbered successively and joined by straight lines to produce trajectories as shown in figures 6.1 and 6.2. The notebook page featured in figure 6.2 indicates that this particular set of observations took place one undated afternoon between 1:00 p.m. and 1:30 p.m. Altogether, 950 observations were made using this particular emulsion, with each observation corresponding to one curve segment.¹⁵

Theory and Experiment

Brownian motion, the irregular and perpetual motion of small particles suspended in a liquid or a gas, varies in intensity as a function of the viscosity and temperature of the medium, but also of the size of the particles: the smaller the particle, the greater the motion. It accordingly affects microscopic and submicroscopic particles particularly strongly (as such it is well known to microscopists for interfering with microorganisms' proper motions). Brownian motion presented an opening for Perrin as a phenomenon that might be enrolled in his advocacy of atomism. In previous years several scientists, including Aimé Cotton and Pierre Curie's close friend the physicist Georges Gouy, had suggested that the Brownian motion of microscopic particles was a perceptible consequence of molecular agitation in fluids, and therefore that the phenomenon could be interpreted as empirical evidence in favor of atomism and the kinetic theory (which supposed liquids and gases to be made up of very small, hard spheres, or atoms).¹⁶ However, as even the supporters of atomism recognized, existing measurements of the Brownian motion of particles did not correspond, by far, to the values predicted by the kinetic theory, in turn casting doubt on the atomic hypothesis.

Perrin's project explicitly aimed at producing experimental evidence in favor of the molecular-kinetic interpretation of Brownian motion. For this, he relied on new methods for measuring this motion that offered hope to bring observation in line with the values predicted by the kinetic theory. The temporal development of Perrin's Brownian motion work, as recorded in his laboratory notebooks, makes clear that Einstein's theory was a primary resource for developing his own project. The first entry in his first notebook, begun around March–April 1908, begins: "Langevin-Einstein hypothesis: each granule is assimilable to molecule (same $\sqrt{mv^2}$)." Further, Per-

rin consigned “requires experimental verification,” outlining first ideas for producing emulsions of same-sized grains by centrifugation and measuring experimentally Avogadro’s number, N the number of molecules in a mole. Then Perrin noted: “Thereafter, nothing comes in the way of verifying Einstein’s formula

$$\sqrt{\Delta_x^2} = \sqrt{t} \sqrt{\frac{RT}{N} \frac{1}{3\pi\mu a}}$$

(a radius of the granule μ viscosity), after which the application of this formula enables a to be obtained for *any* arbitrary granule, followed from moment to moment.¹⁷

The reference to his close friend the physicist Paul Langevin on the first page of the notebook is indicative of the latter’s role in attracting Perrin’s attention to Einstein’s work. On 9 March 1908, Langevin had presented a paper to the Académie des Sciences entitled “Sur la théorie du mouvement brownien” that gave Einstein’s formula in a form very close to that appearing in Perrin’s notebook. Langevin also discussed Maryan von Smoluchowski’s publications, and he assessed critically the first attempt at an experimental verification of Einstein’s methods by Swedish physicist The Svedberg.¹⁸ Perrin himself claimed in 1911 that Langevin had first brought Einstein’s investigations to his attention.¹⁹

In his Brownian motion paper of 1905, Einstein had proposed new quantitative methods of measuring Brownian motion and of determining the dimensions of the particles, thus offering novel tools for testing the validity of the kinetic theory. He argued in particular that it was meaningless to measure the instantaneous velocity of individual particles, as previous researchers had done. Instead he proposed to measure their *mean displacement*, suggesting that the mean displacement of a particle on the x axis during an interval of time t should be proportional to the square root of t .²⁰ In Perrin’s formulation of Einstein’s formula above, the mean displacement over a given interval of time $\sqrt{\Delta_x^2}$ can be calculated when R (gas constant), N (Avogadro’s number), T (absolute temperature), μ (the viscosity of the fluid), and a (the radius of the particle) are known; conversely, N or a can be obtained when mean displacement and the other factors are known.²¹ Perrin proposed first to provide an experimental confirmation of Einstein’s formula by calculating the value of N based on the experimental determination of the other factors in the equation. If this N corresponded to the values of N obtained by other methods, the formula could be considered reliable and could in turn be used for determining the size of the suspended grains.

Perrin’s broken lines, then, aimed at determining experimentally the mean

displacement of particles of known radius in a liquid of known viscosity. The squared paper enabled a quick measurement of the length of each segment of his trajectories;²² from there the mean displacement of a particle during successive time intervals could be calculated. Factoring in the constants R and T , Perrin could obtain, for each series of measurements, a value for Avogadro's number N .

Perrin found that the values of N obtained on the basis of 3,000 measured displacements agreed well with other determinations of N he and other scientists, from Lord Rayleigh and J. J. Thomson to the Curies, had made on the basis of different emulsions or different phenomena entirely. This "miracle of concordances" constituted for Perrin decisive evidence of the validity of Einstein's formula, of his method of measuring Brownian motion, and beyond, of the kinetic theory and of molecular reality.²³

It is worth remarking that Perrin's argumentation followed the conventional epistemology of the laboratory sciences of his times in that he put forward his experiments as testing hypotheses derived from theories. As he wrote in 1912: "To this end I searched for a crucial experiment that, by approaching the molecular scale, might give a solid experimental basis to attack or defend the kinetic theories."²⁴ His 1909 *Annales* paper begins with a theoretical discussion before his experimental setup and results are brought up. One should be wary, however, of concluding from these claims that this case illustrates the progressive division of labor between theoreticians and experimentalists that emerged in the early twentieth century, most evidently among the German-speaking physicists. In the French context and in particular in the scientific circles in which Perrin lived and worked, other disciplinary faultlines prevailed. Perrin saw himself primarily as a physical chemist and by no means as an experimental physicist who left theory to more competent colleagues.

Of Photography versus Drawing and the Uses of Statistics in Perfecting Observation

A comparison with contemporary representations of Brownian motion shows that figure 6.1 was by no means the only way in which Brownian motion could be observed and represented, and in fact was quite unusual. Figures 6.3 and 6.4 were, for instance, published in the same year. Maurice de Broglie and Henry Siedentopf, respectively, captured the trajectories of Brownian particles on photographic plates using long exposures. Their images superficially resemble many of the photographs taken by physicists investigating subatomic entities (ions, electrons, α particles) circa 1900, such as photographs of particle tracks in a cloud chamber.

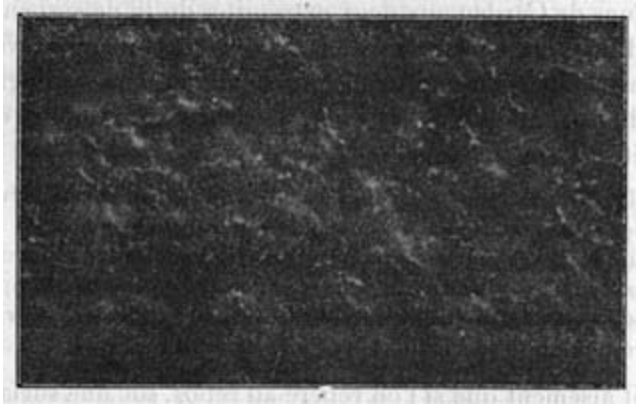


FIGURE 6.3. Long-exposure photographic recording of the Brownian motion of ultramicroscopic tobacco smoke particles. Maurice de Broglie, "Enregistrement photographique des trajectoires browniennes dans les gaz," *Comptes rendus hebdomadaires des séances de l'Académie des sciences* 148 (1909): 1164.

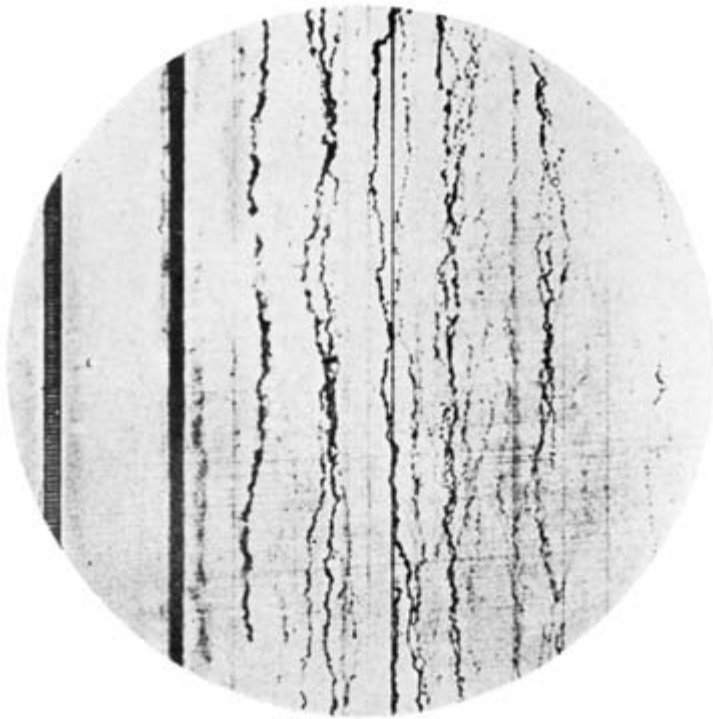


FIGURE 6.4. "Brownsche Molekularbewegung. Momentaufnahme auf fallender Platte mit applanatischem Dunkelfeld-kondensor von Zeiss." Henry Siedentopf, "Über ultramikroskopische Abbildung," *Zeitschrift für Wissenschaftliche Mikroskopie* 26 (1909): plate.

The use of photography for observing and representing Brownian motion brought with it a whole new set of challenges. Perrin occasionally used the technique but found that though photography was less time-consuming and tiring to the eye than a camera lucida, “the eye is more sensitive than the photographic plate with regard to the visibility of very small, pale grains on a background that is nearly as pale.”²⁵ And even when photographic observations could be made, the images obtained did not always give satisfactory results in print, as was especially the case in the observation of the smallest particles, visible only in a particular type of darkfield instrument, the ultramicroscope.²⁶ Maurice de Broglie apologized for the poor quality of figure 6.3: “The imperfection of the typographical reproduction accompanying this note scarcely gives an exact idea of the photographs obtained.”²⁷ Worse still, photographs of Brownian motion were not so easily legible as experimental evidence. In figures 6.3 and 6.4, the particles’ displacements could only be measured by magnifying the photographs, and the Brownian motion had to be distinguished from the overall motion of the gas or solution. In figure 6.4, the Brownian motion of falling particles can be identified as the small deviations on either side of the vertical.

Perrin’s image looks very different. Figure 6.1 is a drawing, not a photograph. It looks more abstract. Even though it depicts the specific trajectories of three specific particles measured at a specific time and place, Perrin stripped from his drawing all elements that might have pointed to a specific experiment. Only the bare essentials remain: three trajectories and a grid. The absence of any indications on the figure suggests that these three lines are simply examples, perhaps even chosen at random, of Perrin’s measurements. They stand for all his other measurements and all the measurements that can be made following his method. The strong presence of the grid in figure 6.1 helps emphasize the quantitative character of the observation. The reproduction of three exemplary trajectories in the publication thus served both to illustrate Perrin’s own technique of measuring Brownian motion and his experimental verification of Einstein’s theory. The unusual appearance of the trajectories as broken lines, underscored by the abstraction of the rendering, was a strong visual marker of the novelty of the method of measuring motion.

An interesting counterpoint to Perrin’s approach was provided by the Marburg-based physicist Max Seddig, who developed around the same time a complex chronophotographic apparatus to test Einstein’s theory of Brownian motion, capturing the position of ultramicroscopic particles at regular intervals of time. To avoid heating the liquid and thereby changing its viscosity during the experiment, Seddig used a stroboscopic technique to illumi-

nate the solution intermittently, timing his photographic camera to open the shutters at exactly the same time. Seddig explicitly put forward his results as being superior to drawings because they were “objectively obtained”: “Following the uncertain results of the subjective methods to date, an objective one should be attempted. As such, only the photographic process could come into question.” Seddig referred here to Felix Exner’s camera lucida drawings on a smoked glass plate of the Brownian motion of particles observed through a microscope (Perrin had not yet published at this point). Such drawings were necessarily unreliable for Seddig because the drawing hand was too slow to follow the particles’ exceedingly rapid motions. His own photographic method, by contrast, supplied a “direct, experimental confirmation” of the kinetic hypothesis.²⁸ Yet Seddig, for reasons unspecified, did not publish any of his photographs, giving only the numerical values he had obtained in the form of a table. Paradoxically, his objective method did not yield images that could be shared with readers, leaving them no other choice than to trust in Seddig’s skills and apparatus.

Perrin used a similar rhetoric, putting forward his own observations of Brownian motion as supplying direct evidence of the existence of atoms. But he diverged from Seddig in arguing that drawing was a perfectly legitimate technique. He admitted freely that marking the positions of the particles by hand introduced a measure of uncertainty: “each time that a grain’s position is marked, a small error is made, analogous to that made when shooting at a target, which itself obeys the law of chance and which has the same effect on the readings as if one overlaid a second Brownian motion over the one under observation.”²⁹ However, this inconvenience was largely compensated by the large number of observations made by Perrin and his collaborators. The striking agreement in the determinations of N made with a range of solutions and colloid particles and in different circumstances showed that small observing errors did not compromise the overall result.

There was also a more fundamental reason for Perrin to trust that the accumulation of measurements compensated for any approximation due to the lack of high-precision recording technology, hinted at in his mention above that observational error could be assimilated to a secondary form of Brownian motion: the fact that Brownian motion could only be investigated using a statistical approach because it was essentially stochastic in character:

Thus appears a profound, eternal property of what we call a *liquid in a state of equilibrium*. This equilibrium only exists in an average manner and for great masses: it is a statistical equilibrium. In reality, the whole fluid is indefinitely and *spontaneously* agitated in movements that are all the more violent the

smaller the portions they concern; the static notion of equilibrium is completely illusory.³⁰

After investigations of radioactive decay (Exner, Curie) and the emission of α particles, Perrin's Brownian motion work constituted one of the early experimental attempts to apply statistics and probability theory to physical systems, and here Perrin followed closely the lead of James Clerk Maxwell and Ludwig Boltzmann.³¹ Brownian motion, considered macroscopically, at the level of the solution, consisted in the minute and random fluctuations around the average state of the fluid. And like the radioactivity experimentalists, Perrin adopted a pragmatic attitude to fluctuations, using knowledge of their existence to develop better methods of measurement.³² His whole strategy aimed at developing experimental and theoretical tools for smoothing out these fluctuations and obtaining average numbers that correspond to the equilibrium state of his emulsions. This statistical approach offered Perrin a means of achieving almost unlimited precision in his observations:

Once this point is well established, one finds in this very equation, to determine the constant N and the constants depending upon it, a method that seems *capable of achieving an unlimited precision*. The preparation of a uniform emulsion and the determination of the values other than N that figure in the equation can indeed be pushed to the desired degree of precision. It is a simple question of patience and of time: nothing limits a priori the exactness of the results, and we can obtain, if we wish, the mass of an atom with the same precision as the mass of the Earth.³³

Once experimental errors were excluded, the greater the number of observations, the closer the average of these measurements would be to the statistical equilibrium and the true value of N . In an experiment that involved measuring the concentration of grains at different levels of his solutions, Perrin made six series of measurements using different solutions and grain sizes. He noted a series of numbers whose average value reached a limit that corresponded to the average frequency of the grains at the level under investigation, remarking that "several thousand readings are necessary if one wants a little precision."³⁴ Just the sixth series of measurements involved counting no less than 11,000 grains using one method, and 13,000 grains using another.

For his Brownian motion work, Perrin was awarded the La Caze Prize in 1914. In their laudatio, the commissioners picked up on this aspect, noting that "Mr. Perrin's method is capable of achieving an infinite precision. It is only a question of patience; it comes down to a numbering of grains and a statistical calculation of averages whose accuracy increases proportionally with the square root of the total number of observations."³⁵ It was

this aspect of Perrin's work, his patience, which most impressed some of his contemporaries — especially given that he was quite forgetful as a person. Much later, in an obituary address Louis de Broglie insisted again on the “tenacity, patience and meticulous attention required by these series of measurements,” which stood in contrast to Perrin's personality, by nature “quite distracted, of a rather impulsive character, such that one might have thought him little suited to carry out a task requiring so much attention and perseverance.”³⁶

With the measurement of the motion of individual particles, this was accomplished by measuring mean displacement. Each displacement, represented in figure 6.1 by a straight segment, was already the result of an averaging process. The three curves of figure 6.1 thus also represented a new statistical approach to the observation of motion. The broken lines stood in stark contrast to the familiar continuous trajectory curves studied since the early days of mechanics. The zig-zag line, even devoid of any indications on the size and nature of the particles or of the interval of time chosen, stood for a new way of conceiving and measuring the motion of individual particles. It was evidence in favor of the statistical approach in the study of the phenomenon of Brownian motion and of events at the molecular level.

Techniques of Observing Motion

But where does figure 6.1 come from? The origin of this form of representation can only be recovered when the broader context in which Perrin worked and the detailed chronology of his investigations are taken into consideration.

Oddly, given that Einstein's mean displacement formula figures in the very early pages of his first notebook, Perrin only undertook experiments to measure the motion of individual particles over a year later, spending the intervening time studying the behavior of systems of particles. Even then, he did not publish figure 6.1 in any of the successive *Comptes rendus* of his work. His first mention of displacement measurements appears in a *Compte rendu* published on 6 September 1909. In the *Annales* paper published just a few weeks later, the image does not appear until page 78, and the discussion of Einstein's theory appears as somewhat of an afterthought.

If, as we have seen, the theoretical work of Einstein and Langevin played an important role in Perrin's project, these do not supply any clues to explain why Perrin turned to observe the motion of individual particles at the time he did and how he came to observe and represent this motion the way he did. For this, we need to turn to another set of resources upon which Perrin drew.

It seems that Perrin's interest in taking up the measurement of the motion

of individual particles had at least one source other than Langevin's work of March 1908. Perrin's very first *Compte rendu* on Brownian motion is dated 11 May 1908. This three-page paper must have struck a chord, since at the following meeting of the academy a week later, no less than two papers were presented that dealt with Brownian motion: the one by Maurice de Broglie discussed above and one by Victor Henri, with the latter explicitly mentioning Perrin and Einstein.³⁷ Henri's report appeared under a *physique biologique* heading, which can be explained by the fact that he worked at the time in the physiology laboratory at the Collège de France (he was himself trained in psychology). Henri expressed doubts about Einstein's formula based on measurements using a sophisticated cinematographical apparatus combined with a microscope.

The preparation is placed in a position precisely horizontal under the microscope. The photographs were taken using a 2mm Zeiss apochromatic objective, with a projection ocular n. 4 and a distance of 24 cm, which gives a magnification of approximately 600 diameters. The light source is a 30-ampere arc lamp; the cinematograph is placed directly above the microscope. The resulting films have twenty images per second and the exposure for each image is $1/320$ of a second; consequently, the interval of time separating two images is equal to $1/20$ of a second.

The selected emulsion was sufficiently diluted such that only about 20 grains appeared in the field of view; in this way they can be located with accuracy and, by determining the position of a grain on a series of successive photographs, the projection of the trajectory described by each grain can be drawn. The figure represents these trajectories for five grains, the successive points corresponding to intervals of $1/20$ of a second, the scale indicating the size of the μ .³⁸

Henri then inserted the image shown in fig. 6.5.

Convinced of the validity of Einstein's formula and keen to disprove Henri, Perrin set one of his students, Chaudesaigues, the task of repeating Henri's experiments. Six months later, on 30 November 1908, Chaudesaigues reported to the academy that his own experiments confirmed Einstein's formula.³⁹ Chaudesaigues had used Perrin's emulsions and a setup combining a microscope and camera lucida. This article featured no images.

The mean displacement measurements presented in Perrin's long article of September 1909, which include both Chaudesaigues' work and the observations carried out with Dabrowski discussed above, had therefore been inspired, at least in part, by Henri's investigations; and so was, clearly, the visualization of Brownian motion that Perrin published at this point for the first time.

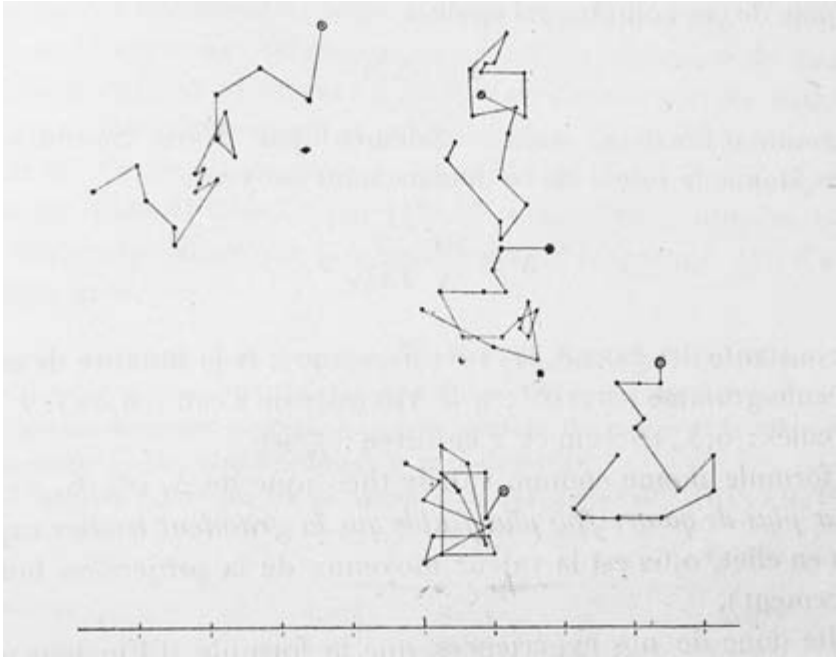


FIGURE 6.5. Victor Henri, “Étude cinématographique des mouvements browniens,” *Comptes rendus hebdomadaires des séances de l’Académie des sciences* 146 (1908): 1025.

Henri, as mentioned, had made his measurements in the physiology laboratory of the Collège de France. This was Etienne-Jules Marey’s laboratory, taken over after his death in 1904 by his former assistant, Charles Albert François-Franck. In 1907, a sophisticated microcinematographical apparatus had been constructed there by Victor Henri and Lucienne Chevroton (assistant and, later, wife of François-Frank) after they visited a Zeiss holiday course on microscopy, ultramicroscopy, and microphotography in Jena.⁴⁰ Upon his return from Jena, Victor Henri promptly integrated this knowledge into his lectures. Chevroton and Henri’s apparatus was also made widely available to researchers for a variety of pursuits,⁴¹ making the years between 1908 and 1910 “an extremely productive one for the development of microcinematography in biology,” according to cinema historian Virgilio Tosi.⁴² Henri’s Brownian motion work was one side investigation using the new technology.

In a direct sense, therefore, Henri’s curves were an extension of the chronophotographic investigation of the motion of humans and animals, applied now to microscopic particles in a move that might have pleased Marey, convinced as he was that his technique was a “universal graphic language” as incontrovertible as geometers’ demonstrations;⁴³ though of course Henri

departed somewhat from the chronophotographic method in that he made a series of photographs, transferring the successive positions onto a synthetic drawing.⁴⁴

In turn, Perrin and Chaudesaigues, wanting to replicate Henri's experiments but not having at their disposal the sophisticated microcinematographic device that was a specialty of François-Frank's laboratory (Perrin revealingly refers to it as an "appareil chronophotographique"), resorted to the simple camera lucida and squared paper to track the motion of the particles. In this way, the chronophotographic *style* of representation was perpetuated while its technological basis was discarded. Henri's, and a fortiori, Perrin's trajectories in a sense *pretended* to be chronophotographic traces.

That no other contemporary investigator of Brownian motion outside Henri and Perrin chose this mode of representation (even those, such as Max Seddig, who independently developed cinema/chronophotographic methods to measure mean displacements) is of course related to their geographical and intellectual proximity and the strength of the Marey school in Paris at the time.⁴⁵ In later years Perrin regularly asked Victor Henri and Jean Comandon (a former student of Henri's who, inspired by Henri's lectures and Brownian motion films, had pioneered the use of the ultramicro-cinematograph for the study of living cells before becoming a professional science filmmaker for the film company Pathé) to prepare films of Brownian motion for showing in public lectures, such as the lecture Perrin gave in Stockholm in 1926 upon receiving the Nobel Prize for physics for his work on Brownian motion.⁴⁶

If Perrin's image was explicitly put forward as an investigation belonging to the physical tradition of studying motion, for example, the trajectories of objects in motion using the laws of mechanics, it could thus equally be considered as belonging to the French physiological tradition of studying animal motion using the graphic method. Almost a century after Robert Brown had identified Brownian motion as distinct from the vital motions of microorganisms, Jean Perrin's visualization of Brownian motion was inspired by physiological techniques of studying the motion of human and animal organisms. Perrin's image brought together the chronophotographic style of representing motion with Einstein's displacement formula to create a new type of image in the physical sciences.

But the application of cinematographic technique to the study of Brownian motion put a new twist on the study of motion through its decomposition. While Marey's technique aimed at decomposing continuous movements into discontinuous snapshots, in the study of Brownian motion, the camera's inherent discontinuity of perception corresponded to the discontinuity of Brownian motion—or rather, to Einstein's way of measuring it, as Scott

Curtis has argued.⁴⁷ In order to measure displacement, it was necessary to measure the position of a given particle at regular intervals of time, ignoring all the intervening motions. For measuring displacements, the camera's "stroboscopic" powers, as one of Perrin's colleagues put it, were perfectly suited (this characterization gives yet another dimension to this episode, connecting it as it does to the history of the strobe discussed in the essay by Jimena Canales in this volume).

Perrin's image of Brownian motion arose out of the convergence in his work of theoretical considerations put forward by Einstein and Langevin, but also of his experience with colloid solutions, learned in part with his teachers Aimé Cotton and Henri Mouton, as well as the chronophotographic techniques of observing and representing motion. The making of Perrin's image and the sources for his novel way of observing Brownian motion can only be recovered by paying close attention to the development in Perrin's own thinking and observing practices, as documented along the way in his laboratory notebooks and the *Comptes rendus* in which he regularly presented his findings, as well as to the immediate intellectual and physical environment in which he lived and worked.

Epilogue

Perrin's work was well received among physicists and chemists in the years following the publication of his *Annales* article of 1909. It was extensively commented upon in chemical and physical journals. German and English translations appeared as booklets in 1910.⁴⁸ Perrin's image was reprinted in longer discussions of his work, the earliest reprint so far identified being a book on the experimental foundations of atomism published in German in 1910 by Werner Mecklenburg, *Die experimentelle Grundlegung der Atomistik*. This is a reprint from the popular journal *Naturwissenschaftlichen Wochenschrift* of 1909–10. Only two images feature in this 143-page booklet, Perrin's figure 6.1 and Henry Siedentopf's figure 6.4. The legend to Perrin's image reads: "The motion of colloid particles according to Perrin."⁴⁹ And of course figure 6.1 features in the publications that appeared on the occasion of Perrin's receiving the Nobel Prize in 1926 and in subsequent commemorative and obituary publications. To this day, Einstein and Perrin's Brownian motion work is taught to physics students and represented in the way Perrin suggested.⁵⁰

In physics, Perrin's image now has only historical relevance as a particular moment in the history of the field, a significant achievement. This is not true of the *theory* of Brownian motion, which was developed in the following de-

cares by physicists and engineers into a more general theory of fluctuations. These investigators studied Brownian motion from the macroscopic perspective, for example, in the form of electronic noise, and using very different techniques of observation and representation.⁵¹

In mathematics also, the theory of Brownian motion remained an ongoing concern and so did, in part, Perrin's representation, in studies that focused on the Brownian motion of individual particles. The American mathematician Norbert Wiener is a central figure in this development, whose Wiener Process is sometimes used as a synonym for Brownian motion. Wiener published a landmark article in 1923 entitled "Differential-Space," in which he developed the mathematical theory of the idealized Brownian motion of a single particle. Wiener begins his paper by noting that mathematicians and physicists are sometimes confronted with similar objects: functions or curves in systems of infinite dimensions. Wiener gives an example from statistical mechanics, the density of a gas obtained from the coordinates and velocities of its molecules. The case he chooses to discuss at length, however, is Brownian motion as theorized by Einstein.⁵²

Wiener writes that the inspiration for this article came from a conversation with French mathematician Paul Lévy. But the choice of Brownian motion as a physical starting point had deeper origins. In his memoirs, Wiener later wrote:

It was at MIT too that my ever-growing interest in the physical aspects of mathematics began to take definite shape. The school buildings overlook the River Charles and command an ever changing skyline of much beauty. The moods of the waters of the river were always delightful to watch. To me as a mathematician and as a physicist they had another meaning as well. How could one bring to a mathematical regularity the study of the mass or ever shifting ripples and waves, for was not the highest destiny of mathematics the discovery of order among disorder? . . . What descriptive language could I use that would portray these clearly visible facts without involving me in the inextricable complexity of a complete description of the water surface? This problem of the waves was clearly one of averaging and statistics, and in this way closely related to the Lebesgue integral which I was studying at the time. Thus, I came to see that the mathematical tool for which I was seeking was one suitable to the description of nature, and I grew ever more aware that it was within nature itself that I must seek the language and the problems of my mathematical investigations.⁵³

Given this interest in mathematical tools that could describe nature, and more specifically the motion of liquids, it is perhaps unsurprising that Wiener read Soddy's translation of Perrin's *Annales* paper. He was particularly struck by

and quotes in “Differential-Space” a passage in which Perrin commented on his Brownian motion curves:

This motion is of such an irregular nature that Perrin says of it: “One realizes from such examples how near the mathematicians are to the truth in refusing, by a logical instinct, to admit the pretended geometrical demonstrations, which are regarded as experimental evidence for the existence of a tangent at each point of a curve.” It hence becomes a matter of interest to the mathematician to discover what are the defining conditions and properties of these particle-paths.⁵⁴

In the same passage Perrin had pointed out that his curves only gave an approximate idea of the “prodigious entanglement” of the real trajectory of individual particles: should the interval of time chosen to mark the particle’s positions be reduced to a second, then each straight line would turn into a curve as complex as the initial one.⁵⁵ Perrin also compared these trajectories to the coast of Brittany, where he spent his holidays every year with, among others, the Langevins, Curies, and Borels. Unlike a map of Brittany, but like its real coast, he pointed out in *Les atomes*, if one zoomed onto the line, each straight segment turned into a broken line and each segment of that broken line also turned into a broken line, and so on ad infinitum. The variation in direction and velocity of the particles was practically infinite. Perrin added that such curves were accordingly continuous but devoid of tangents, corresponding to the nondifferentiable, “pathological” functions that his friend, mathematician Emile Borel, and others had studied since the nineteenth century: “Of course, one cannot either trace a tangent, even approximately, at any point on the trajectory. It is one of those cases where we are reminded of these continuous, nowhere differentiable functions that were wrongly seen as mathematical curiosities, since nature can suggest them just as well as differentiable functions.”⁵⁶

It is very likely that it was Borel himself, with whom Perrin was in daily contact, who inspired these remarks, as they corresponded more closely to Borel’s than to Perrin’s expertise and interests. In the same period, Borel published several articles exploring the relationship between mathematics and statistical mechanics and between physical and mathematical conceptions of infinity, and he frequently brought in Perrin’s Brownian motion experiments.⁵⁷

The mathematicians’ conceptualization of Brownian motion and its relationship to the physical investigations discussed above is another story. But it is already apparent that not only Einstein’s theory but also Perrin’s new way of observing Brownian motion prompted mathematicians to recognize

in his jagged curves a visual counterpart to their nondifferentiable functions, and led them to launch investigations that took into account these functions' newly revealed connection with the natural world. Conversely, the conventions of representing Brownian motion developed by Perrin were transferred to the mathematical representation of idealized Brownian motion and other "random walks." The history of Jean Perrin's Brownian motion work testifies not only to the creative skill involved in making scientific observations, but also to the transformative power of observations once they exist, to their ability to permanently change ways of seeing. This is why to this day Perrin's way of seeing Brownian motion, as encapsulated in figure 6.1, provides a blueprint for the visual representation of Wiener processes and random walks.

Notes

1. For example, Jean Perrin, "La réalité des molécules," *Revue scientifique* 49, n. 2 (1911): 774–84, on 783; idem, *Notice sur les titres et travaux scientifiques* (Paris: Gauthier-Villars, 1918), 77.

2. Jean Perrin, "Mouvement brownien et réalité moléculaire," *Annales de chimie et de physique* ser. 8, 18 (1909): 5–114.

3. Max Born, "Einstein's Statistical Theories," in *Albert Einstein: Philosopher-Scientist*, ed. Paul A. Schilpp (New York: Tudor, 1949), 161–77, on 166; quoted in Stephen G. Brush, "A History of Random Processes. I. Brownian Motion from Brown to Perrin," *Archive for the History of Exact Sciences* 5 (1968): 1–36, on 14–15.

4. Albert Einstein to Jean Perrin, 11 Nov. 1909, in *The Swiss Years: Correspondence, 1902–1914*, vol. 5 of *The Collected Papers of Albert Einstein*, ed. Martin J. Klein, Anne J. Kox, and Robert Schulmann (Princeton: Princeton University Press, 1993), 216.

5. On Einstein's and Perrin's work on Brownian motion, see especially Brush, "A History of Random Processes"; Mary-Jo Nye, *Molecular Reality* (Amsterdam: Elsevier, 1972); John Stachel et al., eds., *The Swiss Years: Writings, 1900–1909*, vol. 2 of *The Collected Papers of Albert Einstein*, ed. Martin J. Klein, Anne J. Kox, and Robert Schulmann (Princeton: Princeton University Press, 1989), esp. 206–22; Roberto Maiocchi, "The Case of Brownian Motion," *British Journal for the History of Science* 23 (1990): 257–83.

6. Henri Poincaré, "Rapport sur la candidature de M. Perrin à la chaire de chimie physique," file F/17/24822, Archives Nationales, Paris.

7. Jean Perrin, *Traité de chimie physique. Les principes* (Paris: Gauthier-Villars, 1903).

8. Jean Perrin, "Les hypothèses moléculaires," *Revue scientifique* 15 (1901): 449–61 (lecture delivered on 16 Feb. 1901 to the students and friends of the University of Paris); see also idem, "La discontinuité de la matière," *Revue du mois* 1 (1906): 323–44, idem, *Notice sur les travaux scientifiques* (Toulouse: Edouard Privat, 1923), 20.

9. See especially Jean Perrin, "Mécanisme de l'électrisation de contact et solutions colloïdales," *Journal de chimie physique* 2 (1904): 601–51, and 3 (1905): 50–110. On Cotton and Mouton's work, see *Notice sur les travaux scientifiques de M. A. Cotton, chargé de cours à l'université de Paris (École Normale Supérieure)* (Paris, 1908); Aimé Cotton, laboratory notebooks 1902–7, box 13, Fonds Aimé Cotton, Physics Library of the École Normale Supérieure, Paris.

10. Perrin, "Mouvement brownien et réalité moléculaire," 79.

11. *Ibid.*, 41.

12. Perrin, *Notice sur les titres et travaux scientifiques*, 63.

13. *Ibid.*, 56.

14. *Ibid.*, 62; Perrin, "Mouvement brownien et réalité moléculaire," 46.

15. Perrin referred to the procedure of viewing the motion of an individual particle and marking its successive positions as "pointés" or "lectures," the resulting segments constituting "observations," that then yielded measurements ("mesures"). Perrin, "Mouvement brownien et réalité moléculaire," 76–80; *idem*, "La réalité des molécules," 783.

16. See, for example, Aimé Cotton, laboratory notebook 1904–5, box 13, Physics Library of the École Normale Supérieure, Paris; Georges Gouy, "Le mouvement brownien et les mouvements moléculaires," *Revue générale des sciences* 6, no. 1 (15 Jan. 1895): 1–7.

17. Jean Perrin, "tube de crookes," laboratory notebook, pages 1–3. Jean Perrin file, Archives of the Académie des Sciences, Paris.

18. Paul Langevin, "Sur la théorie du mouvement brownien," *Comptes rendus hebdomadaires des séances de l'Académie des sciences* (hereafter *CRAS*) 146 (1908): 530–33, on 533.

19. Perrin, "La réalité des molécules," 783.

20. Albert Einstein, *Investigations on the Theory of the Brownian Movement*, trans. A. D. Cooper (London: Methuen, 1926).

21. Albert Einstein, "Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen," *Annalen der Physik* 17 (1905): 549–60.

22. "The grains I had prepared were traced using the camera lucida, which gives horizontal projections of the displacements. Operating with squared paper, one directly obtains the projections on two rectangular axes of the different segments already obtained, but their measurement is unnecessary since the sum of the squares of these projections is equal to the sum of the square of the segments, such that to obtain the average square of the projection on an axis, it is enough to measure these segments one by one, to calculate their square, and to take half of the average of these squares." Perrin, "Mouvement brownien et réalité moléculaire," 76–77.

23. Jean Perrin, *Les atomes* (Paris: Gauthier-Villars, 1913), 289.

24. Jean Perrin, "Rapport sur les preuves de la réalité moléculaire," in *La théorie du rayonnement et des quanta*, ed. Paul Langevin and Maurice de Broglie (Paris: Gauthier-Villars, 1912), 153–253, on 157.

25. Perrin, "Mouvement brownien et réalité moléculaire," 49.

26. On Perrin's use of the ultramicroscope, see Charlotte Bigg, "Evident Atoms: Visuality in Jean Perrin's Brownian Motion Research," *Studies in the History and Philosophy of Science* 39 (2008): 312–22.

27. Maurice de Broglie, "Sur des mesures de mouvements browniens dans les gaz et la charge des particules en suspension," *CRAS* 148 (1909): 1315–18, on 1315.

28. Max Seddig, *Messung der Temperatur-Abhängigkeit der Brownschen Bewegung*, Habilitationsschrift zur Erlangung der Venia legendi der Akademie in Frankfurt a. M. (Frankfurt, 1909), 19, 10, 12 and 32. This argument is echoed by cinema historian Scott Curtis, who claims that Seddig's use of photography "gave substantial evidentiary support to [his] scientific theories." Scott Curtis, "Die kinematographische Methode. Das 'Bewegte Bild' und die Brownsche Bewegung," *montage/av, Zeitschrift für Theorie und Geschichte audiovisueller Kommunikation* 14, no. 2 (2005): 23–43, on 24.

29. Perrin, "Mouvement brownien et réalité moléculaire," 79.
30. *Ibid.*, 10–11.
31. On the history and context of this statistical approach, see Stephen Brush, *The Kind of Motion We Call Heat: A History of the Kinetic Theory of Gases in the Nineteenth Century* (Oxford: North Holland, 1976); Theodore Porter, *The Rise of Statistical Thinking, 1820–1900* (Princeton: Princeton University Press, 1986); Judy Klein, *Statistical Visions in Time: A History of Time Series Analysis, 1662–1938* (Cambridge: Cambridge University Press, 1997), esp. 161–93.
32. Deborah Coen, "Scientists' Errors, Nature's Fluctuations, and the Law of Radioactive Decay, 1899–1926," *Historical Studies in the Physical Sciences* 32 (2000): 179–205. See also Paul Hanle, "Indeterminacy before Heisenberg: The Case of Franz Exner and Erwin Schrödinger," *Historical Studies in the Physical Sciences* 10 (1980): 225–69.
33. Perrin, "Mouvement brownien et réalité moléculaire," 62–63.
34. *Ibid.*, 43.
35. Anon, "Prix L. La Caze," *CRAS* 159 (1914): 867.
36. Louis de Broglie, "La réalité des molécules et l'oeuvre de Jean Perrin," speech read to the Academy of Sciences on 17 December 1945, 13–14. Dossier Jean Perrin, Archives of the Académie des Sciences, Paris.
37. Maurice de Broglie, "Sur l'examen ultramicroscopique des centres chargés en suspension dans les gaz," *CRAS* 146 (1908): 1010–11.
38. Victor Henri, "Étude cinématographique des mouvements browniens," *CRAS* 146 (1908): 1024–26, on 1025.
39. Chaudesaigues, "Le mouvement brownien et la formule d'Einstein," *CRAS* 147 (1908): 1044–46.
40. Lucienne Chevroton, Victor Henri, and André Mayer, "Rapport sur une mission scientifique à Iéna, en vue d'étudier, à l'institut de microscopie et dans les laboratoires de la fondation Carl Zeiss les progrès réalisés dans la technique de la microscopie, de la microphotographie et de l'ultramicroscope," *Nouvelles archives des missions scientifiques* 17 (1908): 1–73. Note that experiments in microcinematography had previously been made at the laboratory, notably by François-Franck, whose setup has much in common with Chevroton and Henri's apparatus. See Thierry Lefebvre, "Contribution à l'histoire de la microcinématographie: De François-Franck à Comandon," *1895* 14 (1993): 35–46.
41. For example, Lucienne Chevroton and Fred Vlès, "La cinématique de la segmentation de l'oeuf et la chronophotographie du mouvement de l'oursin," *CRAS* 149 (1909): 806–9; Lucienne Chevroton, "Dispositif pour les instantanés et la chronophotographie microscopiques. Techniques des prises de vue," *Comptes rendus des séances de la société de biologie* 66 (1909): 340.
42. Virgilio Tosi, *Cinema before Cinema: The Origins of Scientific Cinematography* (London: British Univ. Film & Video, 2005), 179. On Chevroton, Vlès, Comandon, and French biological microcinematography, see Hannah Landecker, "Cellular Features: Microcinematography and Early Film Theory," *Critical Inquiry* 31 (2005): 903–37.
43. Etienne-Jules Marey, *La méthode graphique* (Paris: Masson, 1878), iii. Marey's method was inspired by the German physiologists' self-inscription techniques. See Andreas Mayer, "Autographen des Ganges. Repräsentation und Redressement bewegter Körper im neunzehnten Jahrhundert," in *Kunstmaschinen. Spielräume des Sehens zwischen Wissenschaft und Ästhetik*, ed. Andreas Mayer and Alexandre Métraux (Frankfurt am Main: Fischer, 2005), 101–38; Frederic L. Holmes and Kathryn M. Olesko, "The Images of Precision: Helmholtz and the Graphical

Method in Physiology," in *The Values of Precision*, ed. M. Norton Wise (Princeton: Princeton University Press, 1995), 198–221.

44. Henri, "Étude cinématographique des mouvements browniens," 1025.

45. And perhaps more broadly, to the tradition of graphical and geometrical methods developed by engineers in France after the Revolution, which Marey admired. See Theodore Porter, *Karl Pearson. The Scientific Life in a Statistical Age* (Princeton: Princeton University Press, 2004), 215–48.

46. Perrin first mentioned such a film in 1911. Perrin, "La réalité des molécules," 776. It was perhaps the film made by Comandon and Siedentopf when the latter visited Comandon in 1911, prior to setting up a similar laboratory for ultramicro-cinematography in the Zeiss factory. Jean Comandon, "L'ultramicroscope et la cinématographie," *La presse médicale* 94 (25 Nov. 1909): 841–43. The only extant Brownian motion film by Comandon that could be found is kept in the Musée Albert Kahn in Boulogne near Paris. This film was likely shown by Perrin, as suggested by the film still reproduced in Georges Urbain and Marcel Boll, eds, *La science: ses progrès, ses applications* (Paris: Larousse, 1933–34), 2: 368.

47. Scott Curtis has made this argument about Max Seddig's experiments using cinematography and ultramicroscopy. Curtis, "Die kinematographische Methode."

48. Jean Perrin, *Brownian Movement and Molecular Reality*, trans. Frederick Soddy (London: Taylor and Francis, 1910); and Jean Perrin, *Die Brownsche Bewegung und die wahre Existenz der Moleküle*, trans. J. Donau (Dresden: Theodor Steinkopff, 1910), the latter being a reprint from the *Kolloidchemische Beihefte*.

49. Werner Mecklenburg, *Die experimentelle Grundlegung der Atomistik* (Jena: Gustav Fischer, 1910).

50. See, for example, Thomas William Körner, *Fourier Analysis* (Cambridge: Cambridge University Press, 1989), where figure 1 appears sideways on 44.

51. See Leon Cohen, "The History of Noise, on the 100th Anniversary of Its Birth," *IEEE Signal Processing Magazine* (Nov. 2005): 20–45; Günter Dorferl and Dieter Hoffmann, "Von Albert Einstein bis Norbert Wiener—frühe Ansichten und späte Einsichten zum Phänomen des elektronischen Rauschens," Max Planck Institute for the History of Science preprint 301.

52. Norbert Wiener, "Differential-Space," *Journal of Mathematics and Physics* 58 (1923): 131–74.

53. Wiener quoted in Steve Heims, *John von Neumann and Norbert Wiener* (Cambridge, Mass.: MIT Press, 1980), 68–69.

54. Wiener, "Differential-Space," 133. The quotation is in Perrin, *Brownian Motion*, 64.

55. Perrin, "Mouvement brownien et réalité moléculaire," 80–81.

56. *Ibid.*, 31.

57. Emile Borel, "Le continu mathématique et le continu physique," *Scientia* 6 (1909): 21–25; *idem*, "Les théories moléculaires et les mathématiques," *Revue générale des sciences pures et appliquées* 23 (1912): 842–53; *idem*, "L'infini mathématique et la réalité," *Revue du mois* 18 (1914): 71–84.